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RELATIVISTIC HYDRODYNAMICS AND HEAVY ION REACTIONS

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The use of hydrodynamics to describe the collision of hadronic matter has a long history which dates from work of Fermi [1], Pomeranchuk [2] and Landau [3] in the early fifties. They attempted to describe proton-proton scattering and the concomitant production of pions using statistical and hydrodynamical concepts. Their success encouraged other, later applications to different reactions, both at higher energies as well as for heavier, composite particles. Since this early work, the models have been refined and fresh concepts have been advanced utilizing new ideas from particle physics, quantum chromodynamics and other fields such as astrophysics. This article will very briefly review a few of the varied applications relativistic hydrodynamics has in the area of heavy ion reactions and anti-proton annihilation. A two-fluid model which overcomes certain of the limitations of the usual relativistic hydrodynamics in describing the physical processes and which also avoids the problems with causality associated with the introduction of dissipation into the hydrodynamic equations will be described. We refer the reader to the literature for more detailed descriptions of the application of hydrodynamics to heavy ion reactions [4,5] and hadron-hadron collisions [6,7].

It is not *a priori* apparent that hydrodynamics will be valid for the description of heavy ion reactions. An examination of the conditions necessary for the validity of hydrodynamics indicates that the requirements are only marginally fulfilled. For example, the number of particles which are involved in a heavy ion collision ranges from perhaps only a hundred to a thousand. Hence, the number of degrees of freedom is large compared to one, but relatively small compared to a usual fluid. If one creates a quark-gluon plasma during the collision, then the number of degrees of freedom will increase by at least a factor of three.

There is also the condition that there be sufficient time for the establishment of local, thermal equilibrium; this also is marginally satisfied. A lower limit on the collision time for two heavy ions may be roughly estimated as the nuclear diameter divided by the velocity of light, or about 5×10^{-23} s. Nucleons interact by exchanging pions and it requires about 5×10^{-24} s for two adjacent nucleons to exchange a pion. Since this interaction time is about one-tenth of the total collision time, some degree of local equilibrium will be established. This will be particularly true for central collisions of large nuclei for which the matter in the interior will be confined for longer periods than the above estimate. Further, for moderate bombarding energies, the actual reaction time is around 20×10^{-23} s which is appreciably larger than the simple above argument suggested.

Bondorf and Zimányi [8] have investigated the approach to equilibrium using a time-dependent Boltzmann equation. They concluded that the pion and proton spectra suggest that the momentum distributions are very near their equilibrium values. During sufficiently energetic reactions, new particles such as pions or deltas may be created; such particles are short lived, either being rapidly absorbed in the case of pions, or decaying into a nucleon and a pion. Montvay and Zimányi [9] have investigated whether chemical equilibrium is reached; they conclude that it is not reached although the system is not very far from it.

Finally, it is not unrealistic to treat the nucleons as classical particles for the energy regimes in which we shall be interested. For relativistic nucleons their momenta is greater than 1 GeV/c and their corresponding de Broglie wavelength is 0.4 fm[1] which is less than the radius of a nucleon and much less than that of a nucleus.

The applications of relativistic fluid dynamics to heavy ion reactions have assumed there to be no dissipation. (There is some early work on hadron-hadron reactions by the Russian school which attempted to include the effects of viscosity. This work assumed the validity of the Landau equations and is reviewed by Feinberg [6].) In the Los Alamos effort the three dimensional relativistic Euler equations are solved numerically using the particle-in-cell method developed by Harlow [10,11]. The particle-in-cell method allows calculations in cases of extreme distortion and shear including cases where cavities appear in the fluid. It also allows beautiful graphical representations of the fluid. However, it consumes vast amounts of computer memory.

The equation of state for nuclear matter is unknown; indeed, one of the goals of heavy ion reactions is to investigate the equation of state. Since theoretical calculations of the energy and pressure of nuclear matter as a function of density and

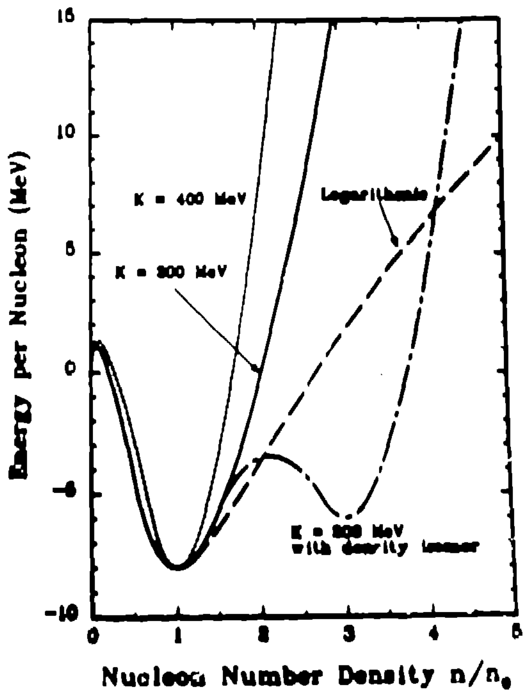

Fig. 1. A plot of the ground state energy per nucleon $E(n)$ as a function of density for four different phenomenological expressions. The quantity K is the compression modulus of nuclear matter, the canonical value of which is 200 MeV.

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[1] It is customary to give masses in energy units; e.g., the mass of a nucleon is 939 MeV (million electron volts), that of a pion is 139 MeV and a delta is 1232 MeV or 1.232 GeV. One usually disregards the mass difference of the neutron and proton and refers to them generically as nucleons. Since the total energy of a particle is the sum of its rest mass and its kinetic energy, the Lorentz contraction factor is $\gamma = 1 + T/m$ where T is the kinetic energy in MeV and m is its rest mass. For the highest energies γ may exceed 100. The unit of length is a fermi (fm) which is 10^{-13} cm.

temperature are quite uncertain, one usually is forced to make assumptions regarding the equation of state. It is normal in nuclear physics to call the energy per nucleon $E(n)$ the equation of state rather than expressing the pressure as a function of temperature and density. The two are of course equivalent since one may obtain the pressure from the usual thermodynamic relation

$$P = \partial E / \partial V)_S \tag{1}$$

where S is the entropy. Examples of some zero-temperature equations of states which have been used are shown in fig. 1. One further usually assumes that the matter obeys a Fermi gas equation of state for non-zero temperature. For densities greater than five times normal nuclear matter density, a number of possible scenarios have been proposed. Currently, the most plausible one is that at sufficiently high densities or temperatures, the nucleons 'melt' and a quark-gluon plasma is formed in which the identities of the individual nucleons is lost and the constituent quarks and gluons are free to briefly roam about the relatively large collision volume.

An example of the time development of a heavy ion reaction is shown in fig. 2 which shows the collision of ^{20}Ne on ^{238}U at 393 MeV/nucleon and two equal mass nuclei at 800 MeV/nucleon. From a knowledge of the velocity vectors of the fluid in each cell at the end of the calculation, one may calculate the double differential cross section $d^2\sigma/dE\,d\Omega$ which may then be compared with experiment. In general the

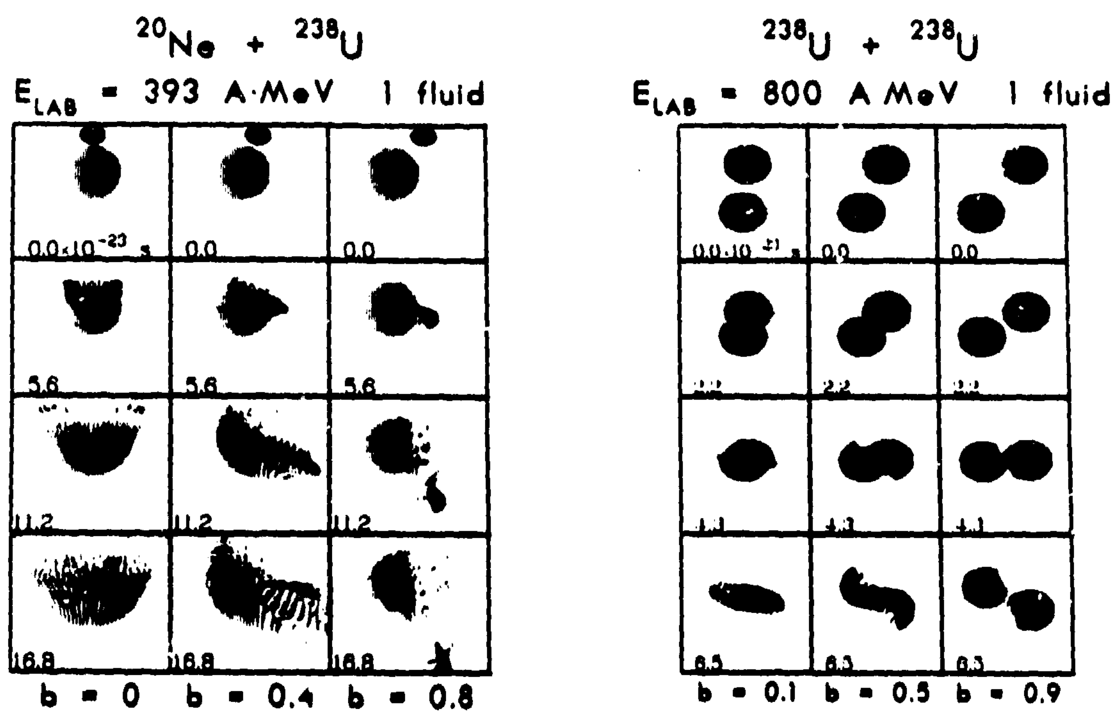

Fig. 2. Matter distributions for 393 MeV/nucleon ^{20}Ne on ^{238}U (left) and two equal-mass nuclei at 162 MeV/nucleon (right) in the center-of-mass (equivalent laboratory energy is 800 Mev/nucleon). Three impact parameters are shown in units of the sum of the radii of the two nuclei. Since the Euler equations are scale invariant, the right figure applies to arbitrary mass nuclei, although the time scale is appropriate only for ^{238}U on ^{238}U.

agreement is satisfactory. In fact hydrodynamic calculations predicted that in certain reactions, the relative incompressibility of nuclear matter would cause the projectile to glance off the target and produce a distinctive signal in the angular distribution. This "sideways" flow was subsequently experimentally observed. For details see ref. 4.

A close examination of fig. 2 will demonstrates that the calculation reproduces the result that for a one-fluid system with no viscosity, the mean-free-path of the matter is zero. (This is more easily seen when the matter from each nucleus is plotted in color as in ref. 5.) This is not a serious problem at low bombarding energies of less than a GeV per nucleon for which the mean free path of a nucleon in the nucleus is much shorter than the nuclear diameter. However, as the energy of the projectile increases, the nucleus becomes more transparent and the assumption of a zero mean-free-path becomes untenable. The effect of non-zero mean-free-paths has been known for some time from high energy proton-nucleus experiments at Fermi Lab and CERN; in these experiments the so-called leading particles punch through the target and carry away a significant amount of the energy of the incident proton. One could simulate to a small extent the effects of a non-zero mean-free-path by introducing viscosity. However, this would introduce all the problems associated with the acausal behaviour as demonstrated by Hiscock and Lindblom [12,13]. In any event, this would be inadequate when the mean-free-paths become so long that some of the nucleons can traverse the entire target and emerge on the far side. Further, as the energies increase it becomes less likely that local thermal equilibrium is instantaneously established at the interface of the two nuclei.

To describe the situation in which large mean-free-paths are involved, a two-fluid model was introduced [14]. To obtain the equations which describe the two-fluid model, each nucleus is assumed to be a fluid which has the identical properties of the fluid representing the other nucleus. When the two fluids collide they are allowed to exchange energy and momentum at a finite rate proportional to the relative velocity of the two nuclei and to the nucleon-nucleon cross section σ_{NN}. Thus, the rate of momentum loss is finite and the two fluids will interpenetrate. The amount of interpenetration is small at low energies for which σ_{NN} is large and increases as σ_{NN} decreases. The Euler equations which ensure particle number conservation remain unchanged, but the equations ensuring energy and momentum conservation must be modified to allow an interchange of these quantities. The changes are in the form of additional terms, the magnitude of which can be estimated from kinetic theory: if one knows the collision rate and the amount of energy and momentum lost in each collision, then the total amount of loss may be found.

The expression for the collision rate is

$$R_{coll} = N_1 N_2 \sigma_{NN} v_{rel}$$

where N_1 and N_2 are the densities of the two fluids and v_{rel} is the relativistic generalization of the relative velocity. The generalized Euler equations for fluid one are

$$\partial_t M_1 + \nabla(v_1 \cdot M_1) = -\nabla P - R_{coll} K (\gamma_1 v_1 - \gamma_2 v_2)/Y \qquad (2)$$

$$\partial_t E_1 + \nabla(v_1 E_1) = -\nabla P - R_{coll} K (\gamma_1 - \gamma_2)/Y \qquad (3)$$

where M_1 and E_1 are the momentum and energy densities of fluid one and Y is the scalar product of the two four-velocities

$Y = (u_1 \cdot u_2)$.

The quantity K determines the amount of energy-momentum loss and is fixed by comparing with high energy nucleon-nucleus reactions. The equations for fluid two are obtained by interchanging the indices 1 and 2.

Unlike the Euler equations, eqs (2) and (3) are not scale invariant; the calculated results will depend the masses of the nuclei involved which is entirely reasonable. A similar consequence occurs if one uses the Navier-Stokes equations. However, unlike the case of the Navier-Stokes equations which introduces dissipation through higher order derivatives of the velocity, the two-fluid model partially achieves the same result by eliminating derivatives in the additional terms.

The additional coupling terms in eqs. (2) and (3) describe the friction between the two nuclei entirely in terms of two-body collisions of the constituent nucleons. It is assumed that the nucleon-nucleon cross section is the free NN cross section σ_{NN} and is independent of density and temperature; this assumption is surely poor at high temperatures and densities. It is further assumed that the Fermi velocities of the nucleons may be ignored. For large relative velocities this is a good approximation (the Fermi velocity at normal nuclear density is approximately 0.27 c); for lower bombarding energies, one must worry about the effects due to the Fermi velocity. For methods which partially take into account the effects of the Fermi velocity, the reader is referred to refs. 5 and 14. In addition both the one-fluid and two-fluid models necessarily omit binding energy effects.

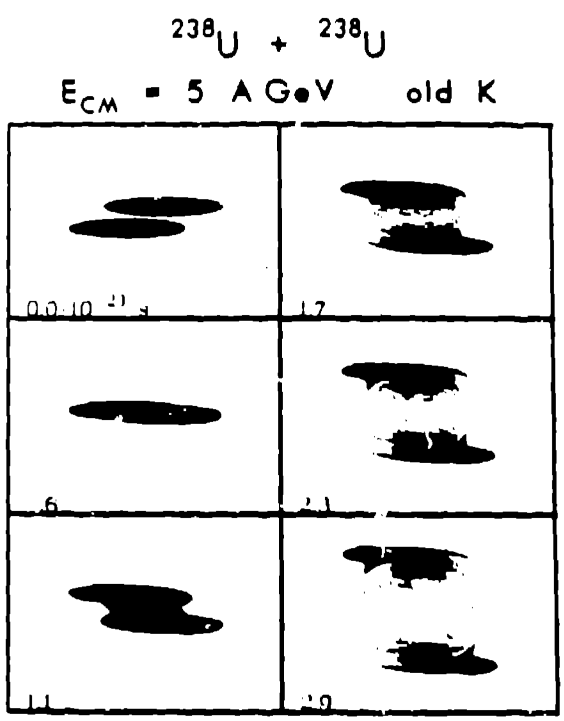

Fig. 3. Matter distributions for ^{238}U on ^{238}U calculated in the center-of-mass system with the two-fluid model. The center-of-mass kinetic energy is 5 GeV/nucleon (equivalent laboratory energy is 73.7 GeV/nucleon) and the impact parameter is 0.3.

In figure 3 results are shown for a collision between two equal mass nuclei, ich having an energy of 5 GeV per nucleon in the center of mass frame. (This energy rresponds to a velocity of 0.987c in the center of mass or a velocity of 0.99992c in e laboratory frame.) The effects of a non-zero mean-free-path are immediately evident. The two nuclei essentially pass through each other, although each nucleus exerts a drag upon the other. In the one-fluid model the matter at the interface of the two nuclei would have come to a halt. All the kinetic energy must be converted into thermal energy. Hence, the one-fluid model can expect to exhibit a larger thermal pressure than does the two-fluid model. This will result in the nuclear matter blowing up and disintegrating sooner.

Experiments have recently begun at CERN which collide 200 GeV/nucleon ^{16}O ions on nuclear targets in a search for signals of a quark-gluon plasma. Similar experiments will soon begin at lower energies at Brookhaven National Laboratory. In all these experiments relativistic hydrodynamics will play an essential role in the interpretation of results.

Another interesting hadronic process which can involve the use of hydrodynamics is the annihilation of anti-protons inside a nucleus. The annihilation of an anti-proton and a proton results in 1.87 GeV being localized for a short time in a very small volume. Thus the energy density is very briefly twice the normal value. If we assume the entire energy appears as thermal energy, then a fireball is generated. If the annihilation occurs at rest or for a very slow anti-proton, no shock wave is generated [15]. Rather, the disturbance propagates outward from the annihilation point via a sound wave. If, however, the anti-proton carries a significant amount of kinetic energy, the situation is much different. The additional kinetic energy drives the hadronic matter into the nucleus and a shock wave is generated [16]. In fig. 4 the matter distribution resulting from an anti-proton annihilation is given. The incoming anti-proton had a kinetic energy of 0.4 GeV. From such interactions one can hope to learn about the nature of nuclear matter in regions of small density but very high temperature. This promising field is still in its infancy.

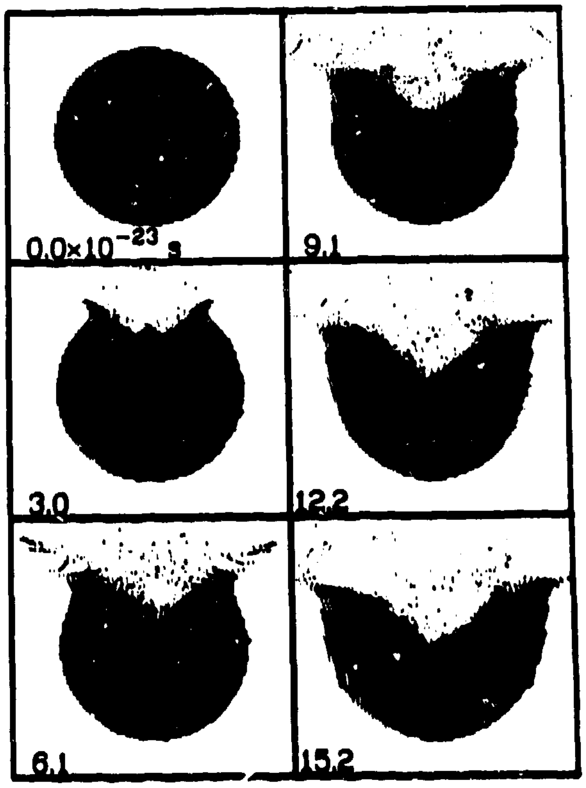

Fig. 4. Nuclear matter distributions resulting from the annihilation of a 400 MeV anti-proton in a nucleus. Only the central 2 fm of the nucleus is plotted.

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